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- USSR -

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The limit equilibrium of homogeneous plates and rotation shells was studied in a number of papers, of which we note [1-4].

Papers [5-7] deal with research on limit equilibrium of plates and shells made of rigid plastic non-homogeneous material. In them the limits of limiting load are determined, originating with a static and kinematic allowable solution of variants.

Works [8, 9] disclose the exact value of the limit load for plates and cylindrical shells made of rigid plastic non-homogeneous material. Below we examine the limit equilibrium of non-homogeneous plates and rotation shells at any piece-wise linear yield conditions.

A parametrical description is given of hypersurface yield for symmetrically loaded rotation shells made of rigid plastic material and complying with the yield condition of a maximal tangential load or with the yield condition of a maximal corrected load. Here for simplicity we assume that the yield limit changes linearly with respect to shell thickness.

As an example, we examine the limit equilibrium of a spherical shell acted upon by an equally distributed internal pressure.

1. Let us examine a symmetrically loaded rotation shell made of rigid plastic material complying to the Tresk yield condition, illustrated by hexagon ABCDEF in fig. 1. This condition has the form

$$\max|\sigma_i - \sigma_j| = k(z, \theta) \qquad (i \neq j = 1, 2) \quad (1.1)$$

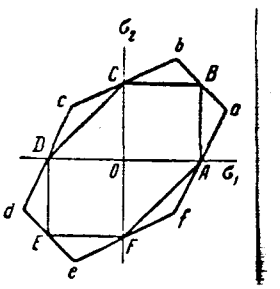

Fig. 1.

Here σ_i, σ_j are the main loads, $\sigma_3 = 0$, and $k(r, \theta)$ is the material's yield limit, changing by an arbitrary laws.

In determining the shell's carrying capacity it is necessary to construct a yield hypersurface in the area of resultant forces N_i and moments M_i (i=1, 2).

In constructing a hypersurface we will use the following system [3]. We examine a plane, the coordinate axes of which ε_1 and ε_2 will (fig. 2). In accordance with the Kirchhoff-Lyav hypotheses

$$\varepsilon_1 = \varepsilon_{10} + z\varkappa_1, \qquad \varepsilon_2 = \varepsilon_{20} + z\varkappa_2 \qquad (1.2)$$

where ε_{10}, $\varkappa_i$ are the main elongation and flexure velocities of the shell's coordinate surface.

It is obvious that $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, in the first quadrant of the plane ε_1, ε_2. In accordance with the associated streaming law, to any point of the first quadrant of

plane ε_1, ε_2 (see fig. 2) corresponds a point lying in angle B of the Tresk hexagon. Analogously, each of the six areas designated by letters in fig. 2 corresponds to the defined angles of the Tresk hexagon (see fig. 1). The sides of the yield hexagon correspond to lines dividing the areas on fig. 2.

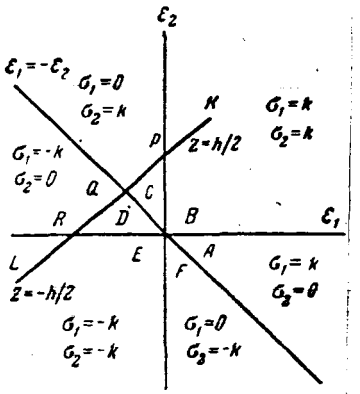

Fig. 2.

It is obvious that the speed of deformation in the direction normal to the coordinate surface of the shell will be pictured as a segment of a line, whose end points correspond to the values $z = \pm h/2$. (Below we will consider that the coordinate surface coincides with the average shell surface. In this case it will be possible to get a very simple parametrical description of hypersurface of yield.

The line segment KL (see fig. 2) can be in different positions. In particular, it can be totally within any area (A, B, C etc.), or it can cross one, two or three lines dividing these areas. We introduce the designation

$$hp = -\frac{\varepsilon_{10}}{\varkappa_1}, \qquad hr = -\frac{\varepsilon_{20}}{\varkappa_2}, \qquad hq = -\frac{\varepsilon_{10}+\varepsilon_{20}}{\varkappa_1+\varkappa_2} \qquad (1.3)$$

Here hp, hr, hq are values of z at which line KL crosses axes ε_2, ε_1, and $\varepsilon_1 = -\varepsilon_2$. We designate the intersections of line KL with axes ε_2, ε_1, $\varepsilon_1 = -\varepsilon_2$ correspondingly by P, R, Q.

Let us examine a more general case, when the line crosses three axes simultaneously and is positioned as shown in fig. 2. We find the resultant forces and moments in this case.

In correspondence with the position of the line we expand the integration limits in the following way:

$$N_i = \int_{-1/2h}^{1/2h} \sigma_i \, dz = \int_{hp}^{1/2h} \sigma_i \, dz + \int_{hq}^{hp} \sigma_i \, dz + \int_{hr}^{hq} \sigma_i \, dz + \int_{-1/2h}^{hr} \sigma_i \, dz$$
$$M_i = \int_{-1/2h}^{1/2h} \sigma_i z \, dz = \int_{hp}^{1/2h} \sigma_i z \, dz + \int_{hq}^{hp} \sigma_i z \, dz + \int_{hr}^{hq} \sigma_i z \, dz + \int_{-1/2h}^{hr} \sigma_i z \, dz \qquad (1.4)$$

Here it is assumed that to point K of line KL (see fig. 2) corresponds the value $z = \frac{1}{2}h$, and to point L the value $z = -\frac{1}{2}h$.

Beginning with values σ_i in corresponding areas, we get the following value for resultant forces N_1 and N_2:

$$N_1 = \int_{hp}^{1/2h} k(z, \theta) \, dz - \int_{hr}^{hq} k(z, \theta) \, dz - \int_{-1/2h}^{hr} k(z, \theta) \, dz$$
$$N_2 = \int_{hp}^{1/2h} k(z, \theta) \, dz + \int_{hq}^{hp} k(z, \theta) \, dz - \int_{-1/2h}^{hr} k(z, \theta) \, dz \qquad (1.5)$$

The resultant moments are found analogously.

In the case being examined, points P, Q, R are different and are within the limits of interval $-\frac{1}{2}h \leq Z \leq \frac{1}{2}h$, whence point Q lies between points P and R.

In the two other cases of simultaneous crossing of three axes by line KL, points P,Q,R will also be different; they will remain within the limits of interval $-\frac{1}{2}h \leq Z \leq \frac{1}{2}h$, but P and R will be the middle points.

Below a parametrical description is given of the bounds of hypersurface yield, corresponding to the three examined cases of the line's position. Here it is considered that the yield limit with single axis stress-strain changes only with respect to shell thickness by the law

$$k(z) = k_0 + 2k_1 z / h \qquad (1.6)$$

Here k_0 and k_1 are certain constant coefficients depending in the general case on the parameters characterizing

the non-homogeneity of the plastic material. For the given case the parametrical description of hypersurface yield has the form

$$
\begin{array}{l}
P \\
\pm n_1 = -(p+q) + 3\gamma\,(1 - 2p^2 - 2q^2) \\
\pm n_2 = -(q-r) - 6\gamma\,(q^2 - r^2) \\
\pm m_1 = (1 - 2p^2 - 2q^2) - 16\gamma\,(p^2 + q^2) \\
\pm m_2 = 2\,(r^2 - q^2) + 16\gamma\,(r^2 - q^2) \\
Q \\
\pm n_1 = -(p+q) + 3\gamma\,(1 - 2p^2 - 2q^2) \\
\pm n_2 = -(q+r) + 3\gamma\,(1 - 2q^2 - 2r^2) \\
\pm m_1 = (1 - 2p^2 - 2q^2) - 16\gamma\,(p^2 + q^2) \\
\pm m_2 = (1 - 2q^2 - 2r^2) - 16\gamma\,(q^2 + r^2) \\
R \\
\pm n_1 = -(q-p) - 6\gamma\,(q^2 - p^2) \\
\pm n_2 = -(q+r) + 3\gamma\,(1 - 2q^2 - 2r^2) \\
\pm m_1 = 2\,(p^2 - q^2) + 16\gamma\,(p^2 - q^2) \\
\pm m_2 = (1 - 2q^2 - 2r^2) - 16\gamma\,(q^2 + r^2)
\end{array}
$$

In relationships (1.7) the following designations are used:

$$n_i = N_i / N_0,\ m_i = M_i / M_0,\ N_0 = k_0 h,\ M_0 = k_0 h^2 / 4,\ \gamma = k_1 / 6k_0$$

The letters above (P, Q, R) indicate which of the points will be average for the given parametrical dependencies.

The associated yield law for all cases corresponding to relationships (1.7) will have the form

$$N_0\varepsilon_{10} : N_0\varepsilon_{20} : M_0\varkappa_1 : M_0\varkappa_2 = +4p\,(r-q) : +4r\,(q-p) : (q-r) : (p-q) \qquad (1.8)$$

The bounds of hypersurface yield for non-homogeneous rotation shells, determined by relationships (1.7), are similar to corresponding bounds of hypersurface yield for homogeneous shells [2].

The case in which line KL transects two of three axes ε_1, ε_2, $\varepsilon_1 = -\varepsilon_2$ corresponds to the situation where one of the points, for example point R for the first position of line KL in fig. 3, crosses the boundary $z = -\frac{1}{2}h$ and is outside of the interval

In the case where the line crosses only one axis, two of three points P,Q,R are outside the limits of interval $-\frac{1}{2}h \leqslant Z \leqslant \frac{1}{2}h$ -- for example, points Q and R for the second position of line KL.

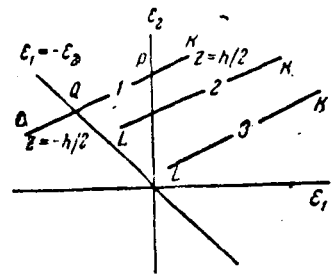

Fig. 3.

If line KL is completely inside one of the areas -- for example, the third position of line KL -- then all three points are outside the limits of interval $-\frac{1}{2}h \leqslant z \leqslant \frac{1}{2}h$. Calculations show that in all of these cases we do not get new points of hypersurface yield. In order to find corresponding values n_i, m_i in all three cases, it is possible to use relationships (1.7), where one (in the first case), two (in the second case) and three (in the third case) parameters p, q, r must be placed equal to $\pm \frac{1}{2}$. Here in the first and second cases we get the minimum of hypersurface yield, and in the third case its peak.

The case in which line KL coincides with one of the axes corresponds to the equating of two parameters, while the third parameter remains unknown. For example, with superposition of the line with axis ε_2 we have hr=hq, and parameter hp is unknown. Eliminating parameter q=r from relationships/m_2, n_2, we get an equation of the bounds of hypersurface yield and a streaming law corresponding to it in the following form:

$$m_2 = \pm\left[1 + \frac{B(3-2\sqrt{B})-1}{108\gamma^2}\right] \quad \text{при } Q=R\ (B=1-12\gamma(n_2-3\gamma))$$
$$\left\{N_0e_{10} : N_0e_{20} : M_0\varkappa_1 : M_0\varkappa_2 = 0 : \pm\frac{1}{3\gamma}(1-\sqrt{B}) : 0 : 1\right\} \qquad (1.9)$$

[Legend]: a) at.

The two other cases of superposition of the line with axes ε_1 and $\varepsilon_2 = -\varepsilon_2$ give the following equation of the bounds of hypersurface yield

$$\begin{gathered} m_1 = \pm\left[1 + \frac{C(3-2\sqrt{C})-1}{108\gamma^2}\right] \quad \text{при } P = Q \\ \left\{N_0\varepsilon_{10} : N_0\varepsilon_{20} : M_0\varkappa_1 : M_0\varkappa_2 = \pm\frac{1}{3\gamma}(1-\sqrt{C}) : 0 : 1 : 0\right\} \\ m_2 - m_1 = \pm\left[1 + \frac{A(3-2\sqrt{A})-1}{108\gamma^2}\right] \quad \text{при } P = R \\ \left\{N_0\varepsilon_{10} : N_0\varepsilon_{20} : M_0\varkappa_1 : M_0\varkappa_2 = \mp\frac{1}{3\gamma}(1-\sqrt{A}) : \pm\frac{1}{3\gamma}(1-\sqrt{A}) : -1 : 1\right. \end{gathered} \tag{1.10}$$

[Legend]: a) at.

Here the magnitudes of A and C are determined by the expressions

$$A = 1 - 12\gamma\,(n_2 - n_1 - 3\gamma), \qquad C = 1 - 12\gamma\,(n_1 - 3\gamma) \tag{1.11}$$

The corresponding hypersurface bounds for a non-homogeneous shell are distorted in comparison with the bounds for a homogeneous shell [2].

The case where the line passes through the origin of coordinates corresponds to the superposition of all three parameters by shell thickness. In this case, in calculation of forces and moments depending upon through which areas the line KL will pass, it follows that we use one of the sets of relationships (1.7), were we must place p=q=r.

It is easy to see that if parameter $\gamma \to 0$, then relationships (1.7), (1.9), (1.10), defining the bounds of hypersurface yield for non-homogeneous rotation shells, convert into analogous relationships defining the bounds of hypersurface yield for homogeneous shells [2]. Using the ideas of paper [4], we can get the parametrical dependencies defining hypersurface yield for any piece-wise linear yield condition.

Let us examine, for example, the condition of maximal corrected load, illustrated on fig. 1 by the hexagon abcdef.

$$\max\{|\sigma_1 - \sigma|, |\sigma_2 - \sigma|, |\sigma_3 - \sigma|\} = k(z, \theta) \qquad (\sigma = {}^1/_3(\sigma_1 + \sigma_2 + \sigma_3)) \tag{1.12}$$

We introduce parameters hp, hq, hr in the following way,

$$hp = -\frac{\gamma_{10}}{\mu_1}, \qquad hr = -\frac{\gamma_{20}}{\mu_2}, \qquad hq = -\frac{\gamma_{10}+\gamma_{20}}{\mu_1+\mu_2} \tag{1.13}$$

where γ_{i0} and μ_i are defined by the main deformation and flexure velocities of the average surface by the formulae

$$\gamma_{10} = \varepsilon_{10} + 2\varepsilon_{20},\ \gamma_{20} = -(2\varepsilon_{10} + \varepsilon_{20}),\ \mu_1 = \varkappa_1 + 2\varkappa_2,\ \mu_2 = -(2\varkappa_1 + \varkappa_2) \tag{1.14}$$

Thus, for the case where points P, Q, R are different by thickness of the shell, we get the following parametrical expression of surface yield:

$$\begin{aligned}
&P\\
&\pm n_1 = -p + q - 2r + 3\gamma(1 - 2p^2 + 2q^2 - 4r^2)\\
&\pm n_2 = -2p - q - r + 6\gamma(1 - 2p^2 - q^2 - r^2)\\
&\pm m_1 = 1 - 2p^2 + 2q^2 - 4r^2 - 16\gamma(p^3 + 2r^3 - q^3)\\
&\pm m_2 = 2 - 4p^2 - 2r^2 - 2q^2 - 16\gamma(r^3 + 2p^3 + q^3)\\
&Q\\
&\pm n_1 = -p + q + 2r - 3\gamma(1 + 2p^2 - 2q^2 - 4r^2)\\
&\pm n_2 = -2p - q + r + 3\gamma(1 - 4p^2 - 2q^2 + 2r^2)\\
&\pm m_1 = -1 - 2p^2 + 2q^3 + 4r^2 + 16\gamma(q^3 + 2r^3 - p^3)\\
&\pm m_2 = 1 - 4p^2 - 2q^3 + 2r^3 - 16\gamma(2p^3 + q^3 - r^3)\\
&R\\
&\pm n_1 = p + q + 2r - 6\gamma(1 - q^2 - p^2 - 2r^2)\\
&\pm n_2 = -q + 2p + r - 3\gamma(1 - 4p^2 + 2q^2 - 2r^2)\\
&\pm m_1 = -2 + 2p^2 + 2q^2 + 4r^2 + 16\gamma(p^3 + q^3 + 2r^3)\\
&\pm m_2 = -1 + 4p^2 - 2q^2 + 2r^2 + 16\gamma(2p^3 - q^3 + r^3)
\end{aligned} \tag{1.15}$$

It must be kept in mind that in relationships (1.15) points P, Q, R are points of intersection of line KL with axes

$$\text{с осями } \gamma_2,\ \gamma_1,\ \gamma_1 = -\gamma_2,$$

where γ_i are the magnitudes of slip velocity, determined by the formulae

$$\gamma_1 = \gamma_{10} + \mu_1 z, \qquad \gamma_2 = \gamma_{20} + \mu_2 z, \qquad \gamma_3 = -(\gamma_1 + \gamma_2) \tag{1.16}$$

$$N_0 e_{10} : N_0 \varepsilon_{20} : M_0 \varkappa_1 : M_0 \varkappa_2 = \tag{1.17}$$
$$= 4(2qr - pq - pr) : 4(2pq - qr - pr) : (2p - r - q) : (2r - q - p)$$

Here it is considered that change in the yield limit by thickness of the shell is described by relationship (1.6).

The bounds of hypersurface yield for non-homogeneous shells, determined by relationships (1.15), are similar to corresponding bounds of hypersurface yield for homogeneous shells [4].

In the case of superposition of two points by thickness of the shell, the surface yield and the relationship of the associated streaming law has the form:

at P = R

$$\frac{m_1+m_2}{3}=\pm\left[1+\frac{A^*(3-2\sqrt{A^*})-1}{108\gamma^2}\right],\left\{\pm\frac{1-\sqrt{A^*}}{3\gamma^2}:\pm\frac{1-\sqrt{A^*}}{3\gamma}:\frac{1}{3}:\frac{1}{3}\right\}$$

at Q = R

$$-\frac{2m_1-m_2}{3}=\pm\left[1+\frac{B^*(3-2\sqrt{B^*})-1}{108\gamma^2}\right]$$
$$\left\{\mp\frac{2(1-\sqrt{B^*})}{9\gamma}:\pm\frac{1-\sqrt{B^*}}{9\gamma}:-\frac{2}{3}:\frac{1}{3}\right\} \qquad (1.18)$$

at P = Q

$$-\frac{m_1-2m_2}{3}=\pm\left[1+\frac{C^*(3-2\sqrt{C^*})-1}{108\gamma^2}\right]$$
$$\left\{\mp\frac{1-\sqrt{C^*}}{9\gamma}:\pm\frac{2(1-\sqrt{C^*})}{9\gamma}:-\frac{1}{3}:\frac{2}{3}\right\}$$

In relationships (1.18) the magnitudes of A*, B*, C* are defined as:

$$A^*=1+12\gamma(n_1+n_2+3\gamma),\qquad B^*=1+4\gamma(2n_1-n_2+9\gamma)$$
$$C^*=1+4\gamma(n_1-2n_2+9\gamma) \qquad (1.19)$$

The bounds of hypersurface yield for a non-homogeneous shell, determined by expressions (1.18), are distorted in comparison with corresponding bounds for homogeneous shells [4].

The peaks and limits of hypersurface yield for non-homogeneous shells, the material of which complies with the yield condition of a maximal corrected load, can be obtained by the same means that were used above in application of the Tresk plasticity conditions. In a similar way dependencies

are obtained that determine hypersurface yield for any piecewise linear plasticity condition.

2. Using the results of part 1, we will examine the limit equilibrium of a spherical shell made of material complying with the Tresk yield condition (1.1).

We will consider that the shell is acted upon by an equally distributed internal pressure; the edges of the shell are fastened Fig. 4, describes a spherical shell, half the arc of which has an angle $\theta = \theta_0$; R is the shell radius.

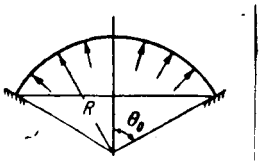

Fig. 4.

To determine the upper limit of the limit pressure we select a velocity field of the form

$$v = 0, \qquad w = \cos\theta_0 - \cos\theta \tag{2.1}$$

The velocity vector (2.1) is directed along the internal normal to the sphere. It becomes zero at the edge, but it does not satisfy the condition of sealing $w' = 0$ at $\theta = \theta_0$; therefore, we will consider the parallel $\theta = \theta_0$ as a hinged periphery. The main elongation and flexure velocities of the average surface of the rotation shell have the form [2]

$$\varepsilon_1 = (v^1 - w)/r_1, \qquad \varepsilon_2 = (v\,\mathrm{ctg}\,\theta - w)/r_2 \tag{2.2}$$
$$\varkappa_1 = -[(v + w')/r_1]'/r_1, \qquad \varkappa_2 = -(v + w')\,\mathrm{ctg}\,\theta/r_1r_2$$

Here r_1 and r_2 are the main radii of flexure of the shell's average surface, and v and w are components of the velocity of shifting in the direction of the meridian and the normal to the shell. Substituting expressions (2.1) in these formulae, we get,

$$\varepsilon_1 = \varepsilon_2 = R^{-1}(\cos\theta - \cos\theta_0), \qquad \varkappa_1 = \varkappa_2 = -R^{-2}\cos\theta \tag{2.3}$$

Parameters p, q, r, determined by relationships (1.3), for the selected velocity field have the form

$$p = q = r = \beta\,(1 - \cos\theta_0/\cos\theta) \qquad (\beta = R/h) \tag{2.4}$$

In the zone near the edges, for values $\theta^* < \theta < \theta_0$, where

$$\cos\theta^* = \frac{\cos\theta_0}{1 - (2\beta)^{-1}} \tag{2.5}$$

parameters $p = q = r < \frac{1}{2}$. For the remaining values of θ parameters $p = q = r > \frac{1}{2}$.

As this follows from the general theory, at $p=q=r > \frac{1}{2}$ we must use relationships (1.7), considering Q to be the central point, in order to calculate the dimensionless resultant of load. Whence in this relationships the parameters are assumed to be equal to $\frac{1}{2}$, and for the force resultants a minus sign is used. From here we have

$$n_1 = n_2 = 1, \qquad m_1 = m_2 = 4\gamma \tag{2.6}$$

The velocity of energy dispersion on a unit area of the shell is equal in this case to

$$d_1 = p_0\,[(1 - \gamma/\beta)\cos\theta - \cos\theta_0] \qquad (p_0 = 2k_0/\beta) \tag{2.7}$$

In the zone near the support, where $p=q=r < \frac{1}{2}$, the load resultants are determined from the same relationships (1.7) with a consideration of general values for p=q=r; therefore, for this zone we will have

$$\begin{aligned} n_1 = n_2 &= 2\beta\left(1 - \frac{\cos\theta_0}{\cos\theta}\right) - 3\gamma\left[1 - 4\beta^2\left(1 - \frac{\cos\theta_0}{\cos\theta}\right)^2\right] \\ m_1 = m_2 &= 4\beta^2\left(1 - \frac{\cos\theta_0}{\cos\theta}\right)^2 - 1 + 32\gamma\beta^3\left(1 - \frac{\cos\theta_0}{\cos\theta}\right)^3 \end{aligned} \tag{2.8}$$

In this way, for the interval $\theta_0 \leqslant \theta \leqslant \theta^*$ the speed of energy dispersion in a unit area will be

$$d_2 = p_0\left[\beta\frac{(\cos\theta - \cos\theta_0)^2}{\cos\theta} + \frac{\cos\theta}{4\beta} - 3\gamma\,(\cos\theta - \cos\theta_0) + 4\gamma\beta^2\frac{(\cos\theta - \cos\theta_0)^2}{\cos^2\theta}\right] \tag{2.9}$$

The speed of energy dispersion in a unit length of the hinged periphery is

$$d_3 = {}^1/_6 p_0 h \sin\theta_0 \qquad (2.10)$$

Integrating expressions (2.7), (2.9) in corresponding limits and summing the obtained results with energy dispersion along the total length of the hinged periphery, we get the total speed of energy dispersion for the whole shell:

$$\Delta/(2\pi R^2) = \int_0^{\theta^*} d_1 \sin\theta d\theta + \int_{\theta^*}^{\theta_0} d_2 \sin\theta d\theta + d_3 \sin\theta_0 R^{-1} \qquad (2.11)$$

If it is assumed that the intensity of internal pressure equals p, then the work of external forces on velocities (2.1) is

$$\frac{w}{2\pi R^2} = \int_0^{\theta_0} p(\cos\theta - \cos\theta_0)\sin\theta d\theta \qquad (2.12)$$

Equating formulae (2.11) and (2.12), we get an expression for the upper limit of the limit pressure

$$p^* = p_0 [1 + F_1(\beta, \theta_0) + \gamma F_2(\beta, \theta_0)] \qquad (2.13)$$

where

$$F_1(\beta, \theta_0) = [\sin^2\theta_0/4\beta - 2\beta \ln(1 - 1/2\beta)\cos^2\theta_0 - (1 + 1/4\beta)\cos^2\theta_0](1-\cos\theta_0)^{-2}$$

$$F_2(\beta, \theta_0) = \left\{\left[\frac{4\beta^3 - 3\beta + 1}{\beta(1 - 1/2\beta)^2} - 6\frac{4\beta^2 - 1}{1 - 1/2\beta} - 24\beta^2 \ln\left(1 - \frac{1}{2\beta}\right) + 20\beta^2 - 4\beta - 3\right]\cos^2\theta_0 - \frac{1}{\beta}\right\}(1 - \cos\theta_0)^{-2} \qquad (2.14)$$

The lower limit for the limit pressure can be obtained from the assumption that the load field at the yield limit is determined by relationships (2.6).

From here we find

$$p_* = p_0 \qquad (2.15)$$

From (2.13) and (2.14) we see that at $\gamma > 0$ the carrying capacity of the shell decreases, at $\gamma < 0$ it increases, and at $\gamma = 0$ from (2.13) a formula is obtained for the limit pressure of paper [2].

Submitted 16 March 1964

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